Some article in Library set.

STANFORD UNIVERSITY LIBRARY SEP 25 1924

The Effective Wave-Length in White-Light Interferometry

A DISSERTATION

PRESENTED TO THE ACADEMIC FACULTY OF THE UNIVERSITY OF VIRGINIA IN CANDIDACY FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

S. C.

BY

FRED ALLISON

Reprinted from
THE ASTROPHYSICAL JOURNAL, Vol. LIX, No. 4, May 1924

3781
V81A
UNCLASSED

STANFORD UNIVERSITY LIBRARIES
WITHDRAWN

The Effective Wave-Length in White-Light Interferometry

A DISSERTATION

PRESENTED TO THE ACADEMIC FACULTY OF THE UNIVERSITY
OF VIRGINIA IN CANDIDACY FOR THE DEGREE
OF DOCTOR OF PHILOSOPHY

BY

FRED ALLISON

Reprinted from
THE ASTROPHYSICAL JOURNAL, Vol. LIX, No. 4, May 1924

THE EFFECTIVE WAVE-LENGTH IN WHITE-LIGHT INTERFEROMETRY

By FRED ALLISON

ABSTRACT

A mathematical theory of the form of the interference fringes from a single source is developed. The theory is then illustrated by some computed instances. It is shown how this theory can be combined with experiment for the purpose of obtaining the effective wave-length for any double star of known black-body temperature. Good agreement is found between the values obtained by the present method and Anderson's experimental value for sunlight.

Practically all measurements with an interferometer involve a knowledge of the wave-length of the source. When white light is used, we are concerned with a certain mean value known as the "effective" wave-length. To reach an exact definition, imagine a perfectly monochromatic source, which at any time may replace the actual source, under such ideal control of the observer that at will he may cause to be emitted any wave-length of the visible spectrum. Suppose that one has under observation some particular phenomenon from a complex source, the first minimum of visibility, for instance, when the interferometer is set on a double star. We now imagine that the observer replaces the double star with the ideal monochromator, and runs the range of wave-lengths until he most nearly matches the phenomenon actually observed from the double star, without changing in any way the adjustment of the instrument. The wave-length thus found is defined as the effective wave-length. If the setting were made on the second minimum of visibility, however, the ideal monochromator would be required to give off a wave-length of different value. Similarly, the effective wave-length for the same source will vary, depending upon the particular phenomenon with which it is associated, as will appear from the more rigorous treatment to follow. The problem, then, is to devise some method for determining the effective wave-length associated with any set of phenomena, the ideal arrangement indicated above, of course, not being at our disposal.

Anderson[1] in a recent paper sets forth the need of the knowledge of the effective wave-length to an accuracy within a few tenths of 1 per cent. He outlines, upon a theoretical basis, a method which was first suggested to him in a communication from Professor C. M. Sparrow, whereby the effective wave-length of any star may be computed, and reports his laboratory determinations of the effective wave-length of sunlight. The present paper presents the results of certain computations and experiments which afford a test of the theory, and indicates some of the precautions that are necessary for work of the highest precision.

Anderson gives an account of earlier contributions to the subject, which need not be repeated. Reference should be made to a former paper of Anderson's,[2] which bears upon the same subject.

In the two papers just mentioned, Anderson has discussed the use of the interferometer in astronomical investigations. A brief recapitulation will help to make more concrete the theory which follows. If the telescope with the interferometer attachment is directed on a double star, the angular separation of whose components is α, the distance $2b$ between the centers of the two apertures of the interferometer is so adjusted that the first minimum of visibility of the fringes is observed. Such an appearance of the fringes is conditioned upon the bright fringes of one component falling upon the dark fringes of the other, the two systems of fringes being out of step by one half-fringe. The condition is realized when

$$\alpha = \frac{\lambda}{4b}. \tag{1}$$

If the source is a star of uniformly luminous circular disk of angular diameter β, the following relation holds for the first minimum of visibility of the fringes

$$\beta = \frac{1.22\,\lambda}{2b}. \tag{2}$$

The result of any interferometer measure (the position angle of a double star excepted) will be expressed in the form

$$\theta = K\lambda, \tag{3}$$

[1] *Astrophysical Journal*, 55, 48, 1922. [2] *Ibid.*, 51, 225, 1920.

where K is determined by the actual observation, and λ depends upon the object observed.

The expressions (1), (2), and (3) are readily derived upon the assumption of a monochromatic source, λ being the wave-length of the source. In the case of an actual star, the λ in each of the three equations is the effective wave-length associated with the phenomenon observed.

GENERAL THEORY

Suppose that we have a distant monochromatic point source from which light is allowed to come through a single aperture so as to produce a diffraction pattern on a distant screen parallel to the plane of the aperture. For our purpose only rectangular and circular apertures need be considered.

For a single rectangular aperture, the resultant amplitude at any point on the screen may be expressed by

$$R_1 = R_0 \frac{\sin(\phi/2)}{\phi/2}, \tag{4}$$

in which ϕ and R_0 represent, respectively, the range of phase and the maximum amplitude.

In the case of a single circular aperture, we may write for the resultant amplitude produced on the screen

$$R_1 = R_0 \frac{J_1(\phi/2)}{\phi/2}, \tag{5}$$

where $J_1(\phi/2)$ is Bessel's function of order unity.

The amplitude due to two similar apertures takes the form

$$R = 2R_1 \cos \frac{\phi b}{2a}, \tag{6}$$

in which a represents half the width of the aperture; b, half the distance separating the two apertures measured along the line joining their centers; and R_1, the amplitude for a single aperture.

For rectangular apertures (6) becomes

$$R = 2R_0 \frac{\sin(\phi/2)}{\phi/2} \cos \frac{\phi b}{2a}. \tag{7}$$

Squaring, we get for the intensity

$$I \equiv I_0 \frac{\sin^2(Aa)}{A^2a^2} \cos^2(Ab), \tag{8}$$

after putting $Aa = \phi/2$.

Similarly, the intensity for two circular apertures is given by

$$I \equiv I_0 \frac{J_1^2(Aa)}{A^2a^2} \cos^2(Ab). \tag{9}$$

Equations (8) and (9) may both be represented by the more general expression

$$I \equiv I_0 F(Aa) \cos^2(Ab), \tag{10}$$

F depending upon the form of aperture.

It remains to consider the extension of the source. The cases of interest are: (*a*) a two-point source, such as a double star; (*b*) an extended source, such as a stellar disk.

For a double-point source having angular separation of 2α and of unequal intensities proportional to I_1 and I_2, and for rectangular apertures, we may write (for small angles) for the intensity corresponding to any angle of diffraction θ, according to equation (10),

$$I(\theta) \equiv I_1 F\left[\frac{2\pi a}{\lambda}(\theta-\alpha)\right] \cos^2 \frac{2\pi b}{\lambda}(\theta-\alpha) + I_2 F\left[\frac{2\pi a}{\lambda}(\theta+\alpha)\right] \cos^2 \frac{2\pi b}{\lambda}(\theta+\alpha). \tag{11}$$

For a uniformly illuminated disk source of angular diameter 2α, the intensity as a function of the angle of incidence i is proportional to $\sqrt{1-\frac{i^2}{\alpha^2}}$, so that equation (10) becomes

$$I(\theta) \equiv I_0 \int_{-\alpha}^{\alpha} \frac{2\pi a}{\lambda}(i+\theta) \cos^2 \frac{2\pi b}{\lambda}(i+\theta) \sqrt{1-\frac{i^2}{\alpha^2}}\, di. \tag{12}$$

Expressions for other types of sources may similarly be formulated, the general form of the expressions being

$$I(\theta) \equiv I_0 \int\int F(i, \theta, \xi) di\, d\xi\,, \tag{13}$$

where ξ is the co-ordinate of an element of the aperture.

In the derivations thus far, we have assumed monochromatic sources. Accordingly, equation (13) expresses the intensity as a function of the co-ordinate of the element of apertures, the angle of incidence, and the angle of diffraction. In order to pass to the actual case of double stars and stellar disks, we must include in the equation a wave-length function. Equations (11), (12), and (13) will then be of the general form

$$I\left(\frac{\theta}{\lambda_0}\right) \equiv I_0 \int\int\int F(\xi, i, \lambda, \theta) d\xi\, di\, d\lambda\,, \tag{14}$$

the integration with respect to λ being taken over the range of visible wave-lengths emitted by the source under consideration.

For a monochromatic source the equation corresponding to (14) is

$$I\frac{\theta}{\lambda'} \equiv I_0 \int\int F(\xi, i, \lambda', \theta) d\xi\, di\,. \tag{15}$$

In the last two equations λ_0 and λ' denote, respectively, the effective wave-length and the monochromatic wave-length. These equations furnish a more rigorous definition of the effective wave-length. It is clearly a function of the slit-widths, the angle of incidence, and the angle of diffraction. Thus, it will not be the same for extended sources, such as stellar disks, as it is for two-point sources, such as the usual type of double star. We may now make the important statement that there is a definite meaning to be attached to the effective wave-length in so far as, for any particular phenomenon of visibility, equations (14) and (15) may be regarded as equivalent.

The problem now is, given the spectral distribution of the source, to find a basis for predicting the interference pattern. This really

amounts to carrying out the computations indicated in equation (14). In the absence of fuller knowledge, the source is taken as a black body, and it is assumed that the phenomena are observed visually by a normal eye through the atmosphere. It will appear from the sequel that these apparently rough assumptions are sufficient for attaining very good agreement with experiment. Three functions of the wave-lengths are thus involved. The spectral distribution E_λ is computed from Planck's formula. The atmospheric transmission A_λ was taken from data supplied by Dr. Anderson (and hence especially applicable to the conditions of his own experiments), and the visibility V_λ was taken from the A.I.S. visibility curve. The product $E_\lambda A_\lambda V_\lambda$ forms what may be called the significant wave-length function, as it gives the spectral distribution of the visual stimulus. The visual stimulus for the interval $d\lambda$ is given by

$$E_\lambda A_\lambda V_\lambda d\lambda \equiv \Phi(\lambda) d\lambda \,. \tag{16}$$

For two infinitely narrow slits and a source emitting light in the interval $d\lambda$ we may write for the visual stimulus at a point x of the screen

$$\Phi(x) \equiv \Phi(\lambda) \cos^2\left(\frac{\pi n x}{2\lambda}\right) d\lambda = \int_0^\infty \frac{c^2 h \lambda^{-5}}{e^{\frac{ch}{k\lambda T}} - 1} A_\lambda V_\lambda \cos^2\left(\frac{\pi n x}{2\lambda}\right) d\lambda \,, \tag{17}$$

where n merely determines our unit of length. n is here taken = 100.

The integral (17) was evaluated by quadratures. The upper and lower limits of integration were 7100 A and 4100 A, at each of which the integral is practically zero. The intervals were 100 A. The computations were carried out for values of x extending from 0 to 330, in intervals of 1 unit; and for the temperature 6800° K., the computations were continued to the value of $x = 780$. From an examination of these extreme values it was found unnecessary to go beyond the value $x = 330$ for the other temperatures. Four sets of computations were made for the temperatures 2000°, 4000°, 6800°, and 40,000° K.

In a similar manner, computations were next made for slits of finite width. This is a still more laborious undertaking because

of the more complicated expression to be integrated. The visual stimulus for finite slits may be put in the form

$$\Phi(x) \equiv \int_0^\infty \frac{c^2 h \lambda^{-5}}{e^{\frac{ch}{K\lambda T}} - 1} A_\lambda V_\lambda \frac{\sin^2\left[\frac{\pi a n x}{2b\lambda}\right]}{\left[\frac{\pi a n x}{2b\lambda}\right]^2} \cos^2\left[\frac{\pi n x}{2\lambda}\right] d\lambda . \qquad (18)$$

The integral was evaluated for slits having the ratios a/b equal to $\frac{1}{4}$ and $\frac{1}{2}$, both for temperature 6800° K.

Some results of interest furnished from the computations for both the infinitely narrow and also the finite apertures are collected in Table I. Certain results of the computations are also shown in Figure 1, *a* and *b*. In these plots the ordinates represent the visual stimulus multiplied by a constant so that for purposes of comparison they all have the same value at the origin, and the abscissae represent the distances on the screen measured from the center of the pattern. In Figure 1*a*, the solid curve is for zero slit-width; the curve represented by dots and dashes for finite slit-width, $a/b = \frac{1}{4}$; the dotted curve for finite slit-width, $a/b = \frac{1}{2}$. The curves of Figure 1*b* correspond to the temperatures 2000°, 4000°, and 40,000°, zero slit-width.

It will be noted from both Table I (p. 218) and the graphs that for infinitely narrow slits the fringes persist over a considerable range, while for the slits of finite width they are rapidly obliterated. No structure in the case of the ratio of 1 to 2 is recognizable beyond the second minimum. There is also to be observed, when the very narrow slits are employed, a progressive increase in the distance between successive maxima and minima, an effect to be expected from theoretical considerations. For the finite slits, however, a forward shifting along the x-axis of the minima and a backward shifting of the maxima were found to exist, a behavior pointed out by Anderson from analytical considerations, as also appears from equation (18). These characteristics of the interference pattern come into special consideration in the study of the pattern resulting from the superposition of two such fringe systems. The positions of maxima and minima recorded in Table I were taken by parabolic interpolation.

FIG. 1

a) Temperature, 6800°: ——, infinitely narrow slits; — · — ·, finite slits ($a/b=\frac{1}{4}$); . . . . ., finite slits ($a/b=\frac{1}{2}$)
b) Infinitely narrow slits: ——, $T=40{,}000°$; . . . . ., $T=4000°$; — · — ·, $T=2000°$

TABLE I

	$T=2000°$ Slit-Width $=0$			$T=4000°$ Slit-Width $=0$			$T=6800°$ Slit-Width $=0$			$T=40{,}000°$ Slit-Width $=0$			$T=6800°$ Finite Slits $a/b=\frac{1}{4}$		
	x	Δ	$\frac{x-x_0}{n}$	x	Δ	$\frac{x-x_0}{n}$	x	Δ	$\frac{x-x_0}{n}$	x	Δ	$\frac{x-x_0}{n}$	x	Δ	$\frac{x-x_0}{n}$
First minimum	5785.6	5785.6	5785.6	5600.7	5600.7	5600.7	5518.8	5518.8	5518.8	5426.5	5426.5	5426.5	5521.6	5521.6	5521.6
First maximum	11593.6	5808.0	5796.8	11212.9	5612.2	5606.4	11051.7	5532.9	5525.8	10861.9	5435.4	5431.0	10848.0	5326.	5424.
Second minimum	17400.7	5807.1	5800.2	16842.1	5629.2	5614.0	16617.2	5565.5	5539.1	16363.4	5501.5	5454.5	16777.5	5930.	5592.
Second maximum	23235.8	5835.1	5809.0	.....	.....	.....	22209.4	5592.2	5552.4	21893.7	5530.3	5473.4	21750.	4972.	5438.
Third minimum	29098.7	5862.9	5819.7	.....	.....	.....	27833.2	5623.8	5566.6	27459.7	5566.0	5491.9	.....	.....	.....
Third maximum	.....	.....	.....	.....	.....	.....	33715.7	5882.5	.....	.....	.....	.....	.....	.....	.....
Fourth minimum	.....	.....	.....	.....	.....	.....	39100.0	.....	.....	.....	.....	.....	.....	.....	.....

	$T=6800°$ Finite Slits $a/b=\frac{1}{2}$		
First minimum	5534.	5534.	5534.
First maximum	10130.	4596.	5065.

x indicates position of maxima and minima.
Δ indicates distance between successive maxima and minima.
$\frac{x-x_0}{n}$ indicates distance from origin to respective maxima and minima divided by 1, 2, 3, . . .

An interesting empirical relation is found to hold between the temperature and the distance from the origin to the first minimum, which we shall designate by λ_x. We may write

$$\lambda_x T + aT + b\lambda_x + c = 0\,, \tag{19}$$

a, b, and c being constants. By solving in the usual manner, using the temperatures 2000°, 6800°, and 40,000° and the corresponding values of λ_x for zero slit-width, we find for a, b, c the respective values -5407.6, 0, 7560. Substituting these values in (19), we get for $T = 4000°$ the value $\lambda_x = 5596.6$, as compared with 5600.7 from Table I. Equation (19) thus takes the simple form:

$$\lambda_x = \frac{7560}{T} + 5407.6\,, \tag{20}$$

an expression which will be seen of value for approximate effective wave-length determinations.

It was also of interest to find from the computed results given by equation (16) the wave-length of maximum luminosity for each of the temperatures. By parabolic interpolation values of the wave-length of maximum luminosity were calculated for the appropriate temperature, the values being 5834.6 A, 5623.8 A, 5524.3 A, 5424.1 A for the respective temperatures 2000°, 4000°, 6800°, 40,000°. The difference between these values, which are sometimes taken for the effective wave-length, and our values for the effective wave-length will be noted from Table II (p. 221). These relations are also shown in Figure 2 (p. 220), in which the visual stimulus of the different wave-lengths is plotted against the wave-length, the ordinates being multiplied by a constant so chosen as to give the same maximum ordinate for each temperature. It is interesting to note the decreasing shift of the maximum in passing from low to high temperatures.

The computations discussed thus far relate to a single point source. For a double star of equal and similar components, we have two fringe systems superposed. The resultant stimulus may be expressed by

$$\Phi\left(x+\frac{c}{2}\right)+\Phi\left(x-\frac{c}{2}\right)=\int_0^\infty \Phi(\lambda)\left[\cos^2\frac{\pi\left(x+\frac{c}{2}\right)n}{2\lambda}+\cos^2\frac{\pi\left(x-\frac{c}{2}\right)n}{2\lambda}\right]d\lambda\,, \tag{21}$$

where c is the relative displacement of the two fringe systems.

The first effort to establish a criterion for minimum visibility was to find by computation what relative shift in the two interference systems would yield a resultant pattern of minimum oscillation. The function $\Phi\left(x+\frac{c}{2}\right)+\Phi\left(x-\frac{c}{2}\right)$ was evaluated for c, equal, respectively, to 5400, 5500, 5600, 5700, and 5800 ($T=6800°$ and

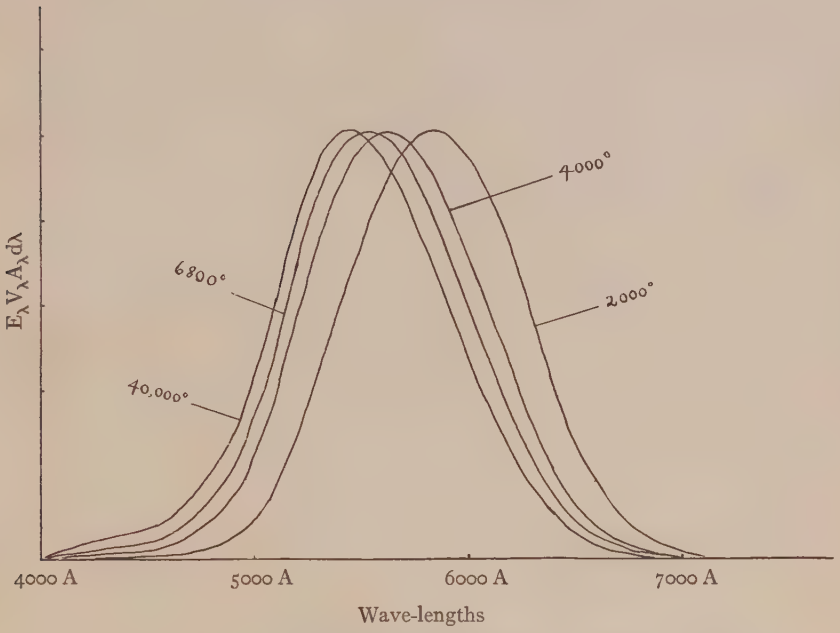


FIG. 2.—Curves showing distribution of luminosity for different temperatures

slit-width = 0). The results are shown graphically in Figure 3. Rayleigh[1] found that in a system of bright and dark bands the structure could be recognized with certainty when a difference of about 3 or 4 per cent existed between the luminosity of the darkest and brightest parts of adjacent bands. According to this, the contrast between the maxima and minima in Figure 3 (p. 222) is more than sufficient for the eye to detect structure, so that the fringes would never quite vanish. The value of c giving the first minimum oscilla-

[1] *Scientific Papers*, 3, 277.

tion was found by interpolation to correspond to an effective wave-length of 5611.3 A. It must be pointed out, however, as appears

TABLE II

EFFECTIVE WAVE-LENGTHS (VISUAL METHOD) INFINITELY NARROW SLITS

Observer	Temperature			
	2000°	4000°	6800°	40,000°
F. A.	5787.2 ±2.6	5610.4 ±2.1	5517.4 ±1.8	5423.2 ±2.4
	5790.7 ±4.1	5615.6 ±1.3	5519.4 ±2.8	5427.6 ±1.4
	5786.4 ±4.4	5608.8 ±1.8	5532.8 ±1.5	5430.9 ±0.7
	5790.4 ±4.4	5596.0 ±3.2	5528.6 ±3.4	5423.6 ±2.1
	5794.0 ±1.1	5605.6 ±3.8	5523.2 ±4.9	5419.4 ±2.1
Mean	5789.7 ±0.9	5607.3 ±2.2	5524.3 ±1.9	5424.9 ±1.3
J. B. B.	5781.8 ±1.6	5617.0 ±3.0	5526.8 ±2.1	5418.8 ±1.6
	5781.2 ±2.1	5616.8 ±1.8	5533.8 ±3.3	5423.9 ±2.7
	5779.4 ±1.4	5615.8 ±3.3	5538.4 ±4.6	5433.8 ±2.1
	5782.4 ±1.5	5610.6 ±1.9	5530.8 ±2.1	5436.2 ±1.6
	5779.6 ±3.9	5612.2 ±2.4	5492.8 ±2.8	5437.8 ±2.3
Mean	5780.9 ±0.4	5614.5 ±0.9	5524.5 ±5.5	5430.1 ±2.5
M. D.	5787.6 ±1.0	5618.8 ±1.5	5537.6 ±3.0	5437.6 ±2.1
	5779.4 ±2.8	5600.0 ±0.7	5505.4 ±5.5	5433.6 ±3.2
	5784.6 ±4.0	5603.4 ±2.9	5520.6 ±7.3	5391.6 ±3.1
	5779.4 ±2.6	5596.0 ±2.4	5528.4 ±2.2	5426.8 ±6.1
	5783.4 ±2.6	5603.2 ±3.1	5511.0 ±1.7	5440.4 ±4.5
Mean	5782.9 ±1.1	5604.3 ±0.7	5520.6 ±3.9	5426.0 ±6.0
L. G. H.	5792.4 ±2.4	5599.2 ±4.5	5537.2 ±1.1	5432.8 ±4.5
	5797.4 ±3.3	5606.2 ±4.1	5507.6 ±2.4	5426.6 ±2.4
	5787.2 ±1.8	5624.2 ±6.0	5532.0 ±3.9	5419.4 ±1.5
	5795.6 ±3.1	5605.8 ±3.6	5528.2 ±2.5	5409.8 ±2.2
	5807.8 ±3.9	5608.0 ±3.2	5528.2 ±3.5	5447.6 ±3.3
Mean	5796.1 ±2.3	5608.7 ±2.8	5526.6 ±3.4	5427.2 ±4.3
W. T. S.	5782.2 ±2.4	5622.4 ±4.3	5521.2 ±3.3	5427.8 ±1.3
	5786.0 ±4.6	5600.2 ±4.7	5520.2 ±1.8	5420.8 ±4.0
	5786.4 ±1.9	5612.8 ±2.2	5534.0 ±2.9	5423.2 ±2.4
	5798.4 ±2.0	5587.4 ±3.6	5510.6 ±3.1	5423.6 ±2.7
	5795.0 ±2.7	5599.8 ±2.6	5506.2 ±2.8	5416.4 ±3.6
Mean	5789.6 ±2.0	5604.5 ±4.2	5518.4 ±2.4	5422.4 ±1.3
Mean for all observers	5787.8 ±1.6	5607.8 ±1.2	5522.9 ±1.0	5426.1 ±0.6

in Figure 3, that this method is open to the criticism that the maximum ratio of adjacent maxima and minima corresponds to different

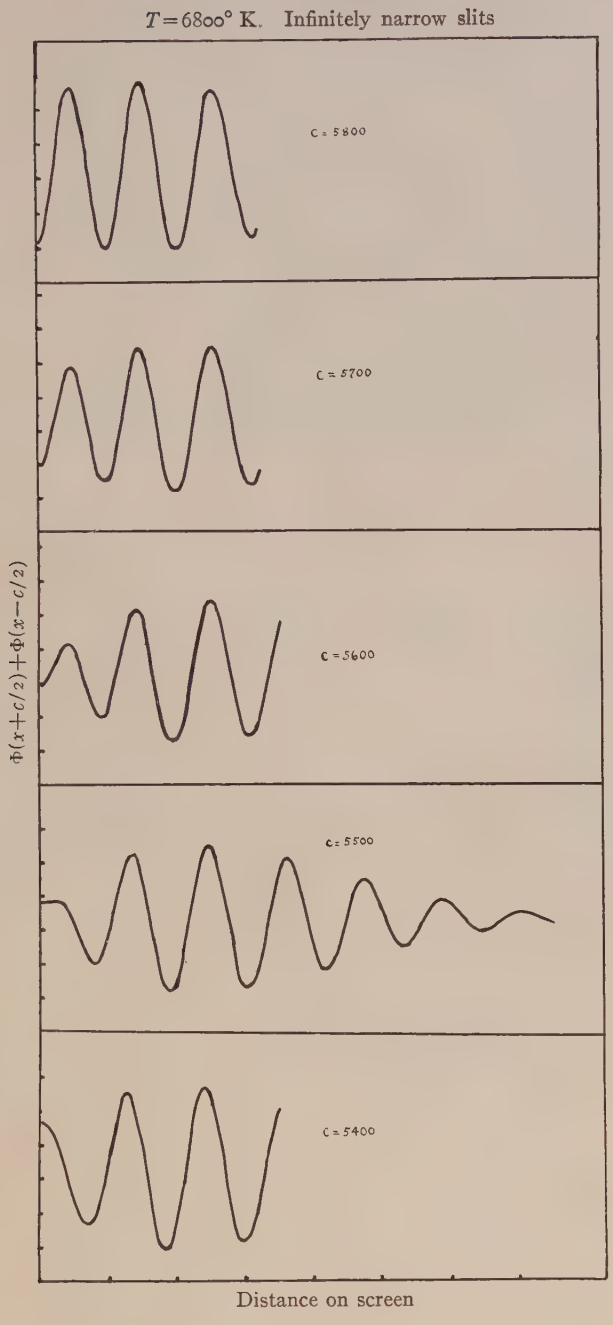

T=6800° K. Infinitely narrow slits
c = 5800
c = 5700
c = 5600
c = 5500
c = 5400
Φ(x+c/2)+Φ(x−c/2)
Distance on screen

FIG. 3

parts of the field of view in the several patterns, and that c will accordingly have values differing with the different parts of the field observed. For instance, if the oscillations studied are those at the center of the field, we find a value of c corresponding to a wave-length 5581.0 A.

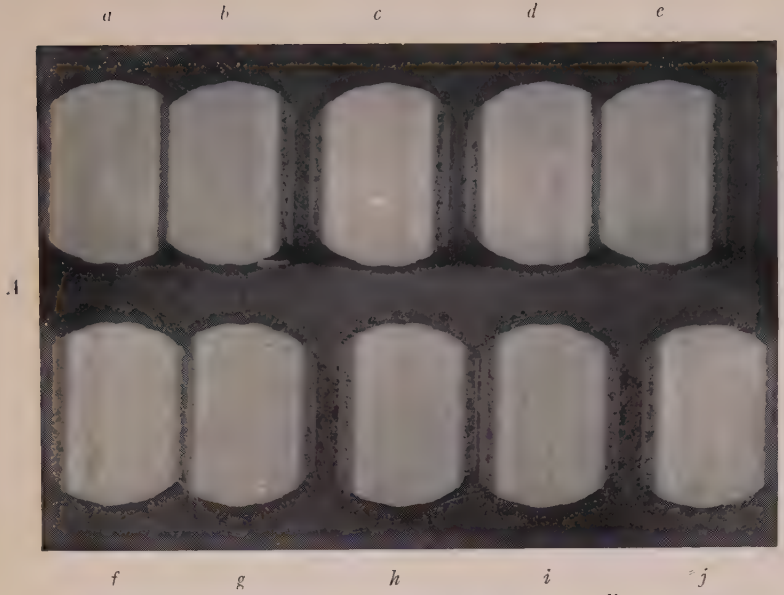
Because of the difficulty of finding a criterion for minimum visibility in the composite curves just discussed, resort was had to a mechanical means of superposing the two patterns in a manner that simulates the phenomena actually observed in an interferometer. The method was much the same as that previously employed by Sparrow.[1] The luminosity curves of Figure 1 were plotted on black paper and mounted on glass slides. Two identical slides were made of each curve, and were mounted one immediately above the other, so that they could be relatively displaced. White light from a uniformly illuminated background sends through the uncovered portions of the slides an amount of luminous energy proportional to that given by equations (17) and (18), and is focused by a cylindrical lens into a system of fringes on the ground glass of a camera. Observations were made visually as the slides were gradually displaced. The patterns could be easily photographed by introducing a plateholder. (See Plates VII and VIII.) Since the eye is more sensitive than the photographic plate to such phenomena, all quantitative results are based on visual settings.

The slides were about 12 by 50 cm. The displacements were measured by means of a vernier, reading directly to 0.01 inch. The cylindrical lens was of about 6-inch focus, and replaced the regular lens of the camera. The distance between the lens and the illuminated slides was about 6 meters.

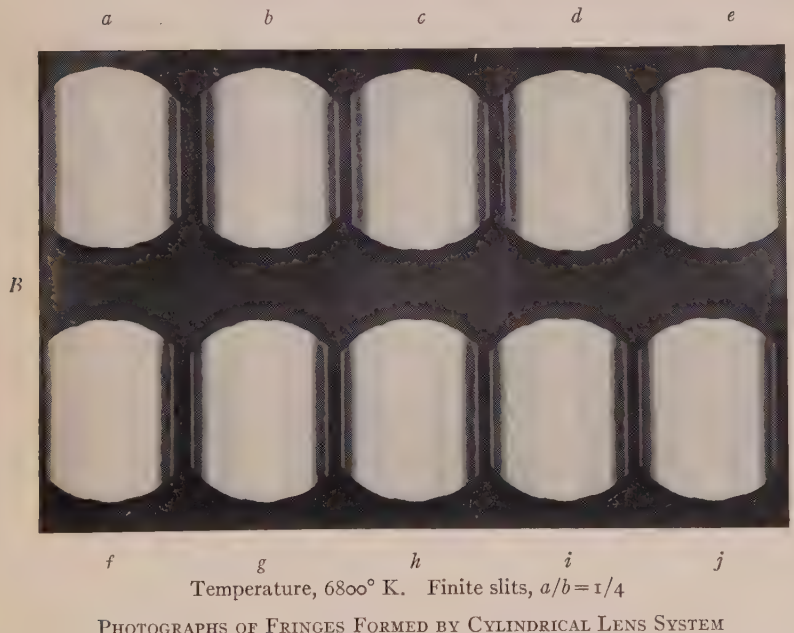
Black paper was finally abandoned in making the patterns in favor of copper foil, in order to eliminate effects due to humidity. The foil was secured to the glass with shellac, after which the curves were plotted with a special device, and the traces were carefully cut. All observations in Table II were made from copper patterns, while those in Table III were from the paper patterns, since the accuracy attainable with finite slits did not seem to warrant the time required to repeat the observations with copper patterns.

[1] *Astrophysical Journal*, 44, 76, 1916.

PLATE VII



Temperature, 6800° K. Infinitely narrow slits.



Temperature, 6800° K. Finite slits, $a/b = 1/4$

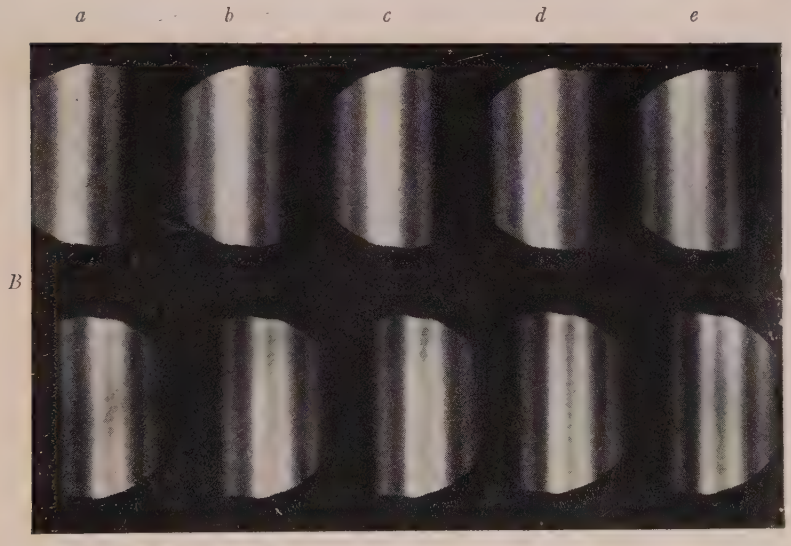
PHOTOGRAPHS OF FRINGES FORMED BY CYLINDRICAL LENS SYSTEM

A.—Ten exposures showing appearance of fringe system in passing through first minimum of visibility in intervals of 0.01 inch (10 A). The successive displacements are indicated by the letters, the minimum being in the *e*–*f* region.

B.—Made under the same conditions as *A*. The failure of the fringes to vanish in the case of the finite slits is more marked than for the very narrow slits.

PLATE VIII

Temperature, 6800° K. Finite slits, $a/b = 1/4$



Temperature, 6800° K. Finite slits, $a/b = 1/2$

PHOTOGRAPHS OF FRINGES FORMED BY CYLINDRICAL LENS SYSTEM

A.—Photographs at slightly greater intervals, 0.05 inch in passing through the first minimum region, which is near E.

B.—Region of first minimum of visibility, at successive displacements of 0.05 inch. Note the failure of the fringes even to approach disappearance.

Tables II and III contain the effective wave-lengths as determined by the visual method. In Table II each value recorded is the mean of five settings. A series of five such sets of observations for each temperature was taken by Professor Hoxton, Messrs. J. B. Ball, M. Dick, W. T. Straley, and the writer. The mean for each temperature for all observers thus represents 125 determinations of the wave-length. A series of five settings occupied usually about fifteen minutes. On account of fatigue it was found advisable to take not more than two or three series of readings the same day. For the very narrow slits (Table II) the accuracy comes well within the bounds set by Anderson.

TABLE III

EFFECTIVE WAVE-LENGTH (VISUAL METHOD)

Observer	FINITE SLITS	
	Temperature 6800°	
	Ratio $a : b = \frac{1}{4}$	Ratio $a : b = \frac{1}{2}$
H. L. A.	5492.8 ±12.	5207.6 ±20.
F. A.	5512.5 ± 4.1	5327.7 ±13.
C. W. B.	5446.8 ± 1.9	5067.9 ± 7.0
J. S. M.	5420.7 ±15.	5156.0 ±40.
Mean	5478.2 ±14.6	5189.8 ±36.6

In Figure 4 the average values of the effective wave-lengths in Table II are plotted against the reciprocal of the temperatures. Circles of radii equal to the probable errors are described about each point. The relation is very nearly linear.

For finite slit-widths the readings, as one would predict from theory, became increasingly discordant for ratios a/b equal to $\frac{1}{4}$ and $\frac{1}{2}$ (Table III). The fringes for ratios employed may scarcely be said to approach disappearance, certainly not for the ratio $\frac{1}{2}$. The degree of the extinction of the fringes for different slit-widths is shown in the accompanying photographs (Plates VII and VIII).

The results show that for wide slit-widths the effective wave-length is somewhat indeterminate, or rather that no such definite quantity exists. For very narrow slits, however, it is found that

there is a definite wave-length associated with the particular phenomenon observed. Since in the recent astronomical investigations the apertures are small, relative to the distance separating them, the results in this paper have practical application. Time did not permit carrying out the computations for ratios smaller than $\frac{1}{4}$. From a consideration of the results before us, however, there seems to be no doubt that for smaller ratios the settings on minimum visibility would be with greater certainty, and the effective wave-length would have a correspondingly more definite value.

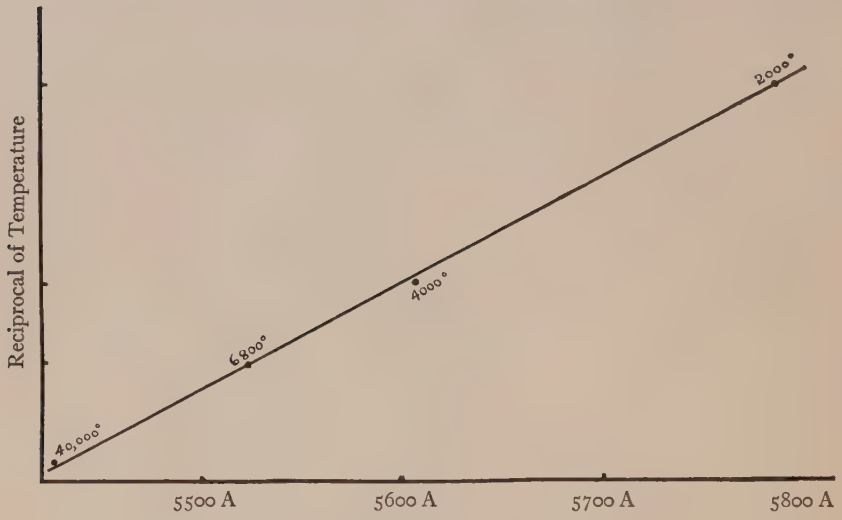


FIG. 4.—Effective wave-lengths (visually determined)

It should be noted that the interference pattern from a single source is obtained from theory, and that the compounding of the two sources is put on an experimental basis.

From the results of our investigations we may now, following Anderson,[1] outline the method of finding the effective wave-length of any double star whose effective temperature corresponding to its spectral distribution is known.

Integral (17) is evaluated for a number of temperatures. These computations are somewhat laborious but they are made once for all. Patterns of copper foil are made from the computed results

[1] *Astrophysical Journal*, 55, 48, 1922.

corresponding to each temperature. These are mounted and relatively displaced until the first minimum of visibility of the fringe system is obtained, one setting to the right and one to the left of the line of centers. The difference between the settings gives the value of the effective wave-length. The average of many such determinations is the value adopted. A graph is then constructed in which these values of the effective wave-length are plotted against the reciprocal of the absolute temperature (Fig. 4), from which the effective wave-length of a star of any effective temperature may at once be found.

To illustrate, suppose that it is desired to find the effective wave-length of an Fo-type star. We look up the effective temperature of the Fo-type star and also the effective temperature of a star of the type of the sun, both temperatures being known from the spectral distribution of the respective stars. We immediately read from our graphs the effective wave-lengths of the Fo-type star and the star of the type of the sun. Denote these, respectively, by λ_1 and λ_2. Anderson[1] has made laboratory measures of the effective wave-length of sunlight by illuminating an artificial double star with sunlight reflected from a freshly polished mirror. Denote this value by λ_0. The effective wave-length λ of the Fo-type star will then be given by

$$\lambda = \lambda_0 + [\lambda_1 - \lambda_2]. \qquad (22)$$

The quantity in brackets is determined by our method. This quantity being relative, the absolute values of λ_1 and λ_2 are not required. For this reason, any error due to the assumption of black-body radiation does not sensibly affect the result. This would mean graphically that the curves representing absolute and relative effective wave-lengths would be approximately parallel, so that differences in the two cases would be for all purposes the same.

It is assumed that the absolute effective wave-length of sunlight is known under the same atmospheric and zenith conditions as those under which the observations are made on the star. Anderson indicates the solution of this problem.

[1] *Astrophysical Journal*, **55**, 48, 1922.

The method applies also to a double star whose components are of unequal intensities and temperatures. Some preliminary observations were taken with patterns representing such components. While it is more difficult to recognize the minimum of visibility, yet the same difficulty will be met when fringes from such a double star are viewed through an interferometer, since the fringes produced by the copper-foil patterns will simulate what appears in the interferometer.

It is worth noting (compare Tables I and II) that within the range of accuracy usually attainable the expression in brackets in equation (22) may be replaced by the distance to the first minimum (λ_x). In the visual determinations the brackets for temperatures 2000°, 4000°, 40,000° take the values 264.9 A, 84.9 A, 96.8 A. For the same temperatures when λ_x is used, the values become 266.8 A, 81.9 A, 92.3 A, changing the value in equation (22) only by the respective amounts +1.9 A, −3.1 A, −4.5 A. The use of λ_x would very materially reduce the labor in obtaining points for the wave-length-temperature curve, since the work of preparing patterns and taking visual observations would be eliminated. Further study is necessary before the statement could be made that such a simplification is possible with heterogeneous components of a double star.

This work was suggested by Professor C. M. Sparrow, to whom I am indebted also for advice and encouragement during the progress of the work. I wish also to thank Professor Hoxton for his interest and encouragement in many ways.

Acknowledgments are due to Professors H. L. Alden and J. S. Miller, Jr., for taking observations; also to Mr. C. W. Bruce, who contributed much time to observing and computing. I desire to thank Mr. A. J. Weed, mechanician, for help and co-operation.

ROUSS PHYSICAL LABORATORY
UNIVERSITY OF VIRGINIA
August 9, 1923